Dealing with the challenges of moving sensor sets for proving final-state observability

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TU Dortmund, November 29, 2022

Abstract

What can one say about the total "energy" $\|x(T)\|_X$ of a system's state on a Banach space X at a final time $T > 0$ given only incomplete observations $\|y(t)\|_X$ for $t \in E \subseteq [0, T]$? Our answer to this question consists of providing a so-called *final-state observability estimate*

$$\|x(T)\|_X \lesssim \int_E \|y(\tau)\|_X \, \mathrm{d}\tau,$$

i.e., estimating the final state's energy by aggregation of observations.

More precisely, we present an extension of the *Lebeau–Robbiano strategy* to the setting of non-autonomous Cauchy problems on general Banach spaces featuring time-dependent families of bounded observation operators. This strategy builds on the existence of a suitable *uncertainty principle* and *dissipation estimate*.

As application, we consider differential operators on $\mathrm{L}^p(\mathbb{R}^d)$ and observation operators $C(t) = \mathbf{1}_{\Omega(t)}$ representing families of *moving sensor sets*. More concretely, we show that a final-state observability estimate holds for the case of an evolution induced by a non-autonomous family of *strongly elliptic differential operators* or non-autonomous *Ornstein–Uhlenbeck operators* subject to a *generalized Kalman rank condition*. For these examples, we will see that the existence of a final-state observability estimate is closely related to geometrical requirements on the family of sensor sets $(\Omega(t))$.

This talk is based on the publication:

C. Bombach, F. Gabel, C. Seifert, and M. Tautenhahn. *Observability for Non-autonomous Systems.* To appear in SIAM Journal on Control and Optimization. arXiv:2203.08469 [math.FA].